THE IMPACT OF CONTRACTUAL TRANSFER RESTRICTIONS AND MICRO LIQUIDITY ON THE DISCOUNT FOR LACK OF MARKETABILITY

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ABSTRACT

The discount for lack of marketability (DLOM) is the difference in value between a public and a comparable private company. However, a growing body of evidence suggests that the DLOM is not only compensation for the lack of marketability (as the name suggests), but that it can also be justified based on other factors that differentiate private and public companies.

Among these DLOM determinants, those that are under the control of private companies remain largely unexplored. Private companies rarely provide information about their internal functioning and, for this reason, the traditional methods that have been employed to study the DLOM often overlook this element in their analysis.

To address this research gap, we created a unique dataset based on judicial decisions in which the court determined the DLOM and then justified the figure by referencing the subject company's specific circumstances. From this dataset, we were able to study and verify the impacts of two critical variables: contractual transfer restrictions on shares and micro liquidity organized by the company.

Ultimately, our study shows that, depending on their nature, contractual transfer restrictions can negatively impact a company's value by approximately 4.7%. The organization of micro liquidity, on the other hand, can generate a significant positive impact of 10% or more.

Keywords: private company – valuation – DLOM – transfer restrictions

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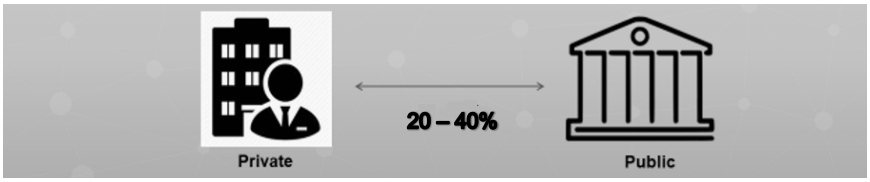
I. INTRODUCTION

Traditional wisdom holds that investors attach a lower value to assets that are not frequently traded.[1] This diminution goes by many different names, but it is commonly referred to as the discount for lack of marketability (DLOM).[2] The DLOM is the difference in value between the stock value of a listed (liquid) company and the stock value of an equivalent private (illiquid) company.[3]

When valuing small private businesses, calculating the DLOM can be a subjective process.[4] More often than not, valuation experts will reduce an entity's estimated marketable value without any economic reasoning.[5] In practice, these experts apply a discount between 20% and 40%.[6]

Illustration 1.



In an effort to objectify these valuation practices, previous research has gone to great lengths to define a standard estimation method and to identify the determining factors of the marketability discount.[7] Nevertheless, these elements remain largely unresolved.[8]

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[1] Axel Buchner, *How Much Can Lack of Marketability Affect Private Equity Fund Values?*, 28 REV. FIN. ECON. 35, 35 (2016); SHANNON P. PRATT & ALICIA V. NICULITA, VALUING A BUSINESS: THE ANALYSIS AND APPRAISAL OF CLOSELY HELD COMPANIES 1, 416 (McGraw Hill, 5th ed. 2007) (stating that entities which are frequently traded are more valuable than those which are not).

[2] The terms "private company discount," "marketability discount" and "illiquidity discount" are used interchangeably in literature. For the purposes of this article, a distinction is made between the private company discount (as a whole) and the marketability or liquidity component. The latter terms will be used interchangeably and in line with the leading literature of the field. *See* Robert Comment, *Business Valuation, DLOM, and Daubert: The Issue of Redundancy*, 29 BUS. VALUATION REV. 83, 83–84 (2010).

[3] Niranjan Chipalkatti et al., *Estimating the Marketability Discounts: A Comparison Between Bid-Ask Spreads, and Longstaff's Upper Bound*, 23 J. APPLIED FIN. 57, 57 (2013).

[4] *Id.*

[5] *Id.*

[6] Ashish Kumar Garg & Kundan Kumar, *Option Pricing Models of Private Equity Valuation: A Comparative Analysis*, 20 IUP J. APPLIED FIN. 28, 28 (2014).

[7] *See* Buchner, *supra* note 1, at 35.

[8] *See* Buchner, *supra* note 1, at 35.

The extant literature unequivocally identifies liquidity and marketability as DLOM determinants.[9] However, the evidence suggests that these factors are not the only drivers behind the discount.[10] In fact, several studies advance other factors while indicating that liquidity has only a modest impact on the DLOM.[11] Therefore, it would be more accurate to refer to the value gap between public and private companies as the "private company discount" rather than as a marketability or liquidity discount.[12]

Additionally, some experts suggest that the discount has little to do with the differences in marketability between public and private companies.[13] In their view, the very notion that the discount is applied for a lack of marketability may be wrong or at least misleading.[14] Still, even these hardliners agree that one of the justifications for applying a valuation discount is the presence of contractual transfer restrictions.[15] In his widely distributed manual, Shannon Pratt echoes this logic, observing that transfer restrictions increase the marketability discount:[16] "Many closely held companies are subject to provisions that severely restrict the rights of the holder to transfer stock. Any provision that limits the right of the holder to transfer the stock would tend to increase the amount of the discount for lack of marketability."[17]

Thus, Pratt's observation lends itself to the following hypothesis: if transfer restrictions influence the discount and are under the control of the company and its shareholders,[18] then the DLOM, and therefore the value, can be partially managed by the company. By the same logic, we can also hypothesize that those positive efforts by a company to enhance liquidity[19] should positively impact the marketability, thereby decreasing the DLOM.[20]

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[9] The terms "marketability" and "liquidity" are used interchangeably, *see* Comment, *supra* note 2, at 84.

[10] Robert Comment, *Revisiting the Illiquidity Discount for Private Companies: A New (and "Skeptical") Restricted-Stock Study*, 24 J. APPLIED CORP. FIN. 80, 91 (2012).

[11] *Id.*

[12] *Id.*

[13] Gilbert E. Matthews, *Private Company Discounts Are Not Caused by Lack of Marketability*, 22 BUS. VALUATION UPDATE 1, 6 (2016).

[14] *Id.*

[15] *Id.* at 3.

[16] *See generally* PRATT & NICULITA, *supra* note 1.

[17] *See* PRATT & NICULITA, *supra* note 1, at 448.

[18] *See* PRATT & NICULITA, *supra* note 1, at 448.

[19] Companies may, for example, enhance liquidity by employing a redemption scheme.

[20] For an illustration of the hypothesized relation, *see* Estate of Marmaduke v. Comm'r. of Internal Revenue, No. 17047-97, 1999 WL 818788, at *12 (T.C. Oct. 14 1999) (stating that while the DLOM for common stock in a company is 30%, the DLOM for the common stock

Now, intrinsically, private companies provide little information about their internal organization, including the transfer restrictions or the availability of micro liquidity. These elements are often based on non-public arrangements, which makes their impact on the DLOM notoriously difficult to study. To overcome this obstacle and to test our hypothesis, we researched an alternative data source: estate and gift tax cases that examine the value of private companies.

In these decisions, we found a wealth of information pertinent to our study. Not only do the courts decide on a DLOM, but, in support of their decisions, they thoroughly examine the inner structure and organization of private companies, including, where applicable, the entity's transfer restrictions as well as the liquidity available to their shareholders.

After analyzing a vast number of cases, we created a unique dataset which enabled us to test our central hypothesis: organizations can, in fact, influence a company's value by including (or omitting) transfer restrictions on shares and by designing a redemption policy (or another form of company-sponsored liquidity).

Illustration 2.

This paper outlines our study and subsequent results, and it is structured as follows: Section II explores the extant literature on the private company discount, including the DLOM factors experts frequently suggest; Section III presents our dataset as well as a taxonomy of transfer restrictions and liquidity; Section IV discusses our methodology and regression results; and Section V summarizes our findings and conclusions.

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held by the ESOP plan is only 20% "because of the liquidity available to the … stock held by the ESOP."). Indeed, ESOP participants had the right to direct the plan to sell shares to the company at the fair market value and thus benefit from a put option. *Id.*

II. LITERATURE REVIEW

A. Conceptual Approaches to Determining the DLOM

The sale of an interest in a closely held entity can be costly, time-consuming, and uncertain. Therefore, in consideration of transaction costs, buyers will demand a lack of marketability discount on the price of an asset equal to the cost of converting the asset to cash.[21] To arrive at a discount estimate, there are three approaches experts may take:[22]

1. Option Model Approach

In the option model approach, experts value the liquidity as a put option.[23] Since the holders of an illiquid asset do not have the option to sell at a time of their choice, the DLOM can be seen as the cost of the option.[24] David Chaffe explains this idea in its purest form: "[i]f one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for those shares."[25]

In the option model approach, the cost of a put option (expressed as a percentage of the value of the underlying restricted asset) serves as a proxy for the DLOM.[26] The models proposed by scholars and practitioners generally retrofit the Black-Scholes-Merton ("BSM") formula,[27] and arrive at high illiquidity discounts (often in the range between 25% and 50%).[28]

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[21] Mukesh Bajaj et al., *Firm Value and Marketability Discounts*, 27 J. CORP. L. 89, 91 (2001).

[22] Aswath Damodaran, *Marketability and Value: Measuring the Illiquidity Discount*, N.Y.U. STERN SCH. BUS. 1, 17 (2005); *see also* Suchismita Mishra, *Liquidity and Asset Pricing Models: A Historical Sketch*, 20 INT'L. J. FIN. 4809 (2008).

[23] Damodaran, *supra* note 22, at 41–42.

[24] Damodaran, *supra* note 22, at 41–42.

[25] David B. H. Chaffe III, *Option Pricing as a Proxy for Discount for Lack of Marketability on Private Company Valuation*, 12 BUS. VALUATION REV. 182, 182 (1993).

[26] *Id.*

[27] *Id.* at 183. The BSM model is a differential equation used to solve for options prices. The model utilizes five inputs: asset price; strike price; interest rates; time to expiration; and volatility. *Id*.

[28] Chaffe, *supra* note 25, at 184 (discussing the put option model); *see also* Chipalkatti et al., *supra* note 3; Francis A. Longstaff, *How Much Can Marketability Affect Security Values?*, 50 J. FIN. 1767 (1995); Robert R. Trout, *Minimum Marketability Discounts*, 22 BUS. VALUATION REV. 124, 124 (2003); John D. Finnerty, *An Average-Strike Put Option Model of the Marketability Discount*, 19 J. DERIVATIVES 53, 53 (2012); Robert Brooks, *A General Option Valuation Approach to Discount for Lack of Marketability*, SSRN (Jan. 15, 2014), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2379007; Menachem Abudy et al., *The*

Among its many critiques, the most significant issue raised against the option model stems from the fact that a put option does not mimic the effects of liquidity.[29] Indeed, liquidity does not give the right to sell at a given strike price; rather, it gives the right to sell at the prevailing market price over a given period. Comparatively, a put option has a different effect because: (1) a put option protects against all downside risk while allowing the investor to maintain the asset's upside potential; and (2) it does not provide for the immediate monetization of the asset.[30]

2. Discount Rate Adjustment

The second approach employed by valuation experts adjusts the required rate of return on an asset to reflect its illiquidity.[31] This approach amounts to adding an illiquidity premium to the discount rate, which in turn leads to a lower value for the same set of expected cash flows.[32] The premium might vary over time and depend upon the market-wide demand for liquidity.

In finance literature, studies that calculate the DLOM based on an increased discount rate in a discounted cash flow (DCF) model are rare.[33] These DCF models generally add a constant illiquidity premium to the discount rate for all illiquid assets.[34]

Effect of Liquidity on Non-Marketable Securities, SSRN (Sept. 26, 2017), https://ssrn.com/abstract=3042136.

[29] *See* Lester Barenbaum et. al, *Determining Lack of Marketability Discounts: Employing an Equity Collar*, 17 J. ENTREPRENEURIAL FIN. 65, 65 (2015).

[30] *Id.* (suggesting that the proceeds of a hedging strategy employing an at-the-money equity collar along with the interest cost of borrowing funds to monetize the underlying asset better captures the cost of providing liquidity to a non-traded asset).

[31] *See* Damodaran, *supra* note 22, at 22.

[32] *See* Damodaran, *supra* note 22, at 35–36; *see also* Mishra, *supra* note 22, at 4814 (explaining the need for an added illiquidity premium: "There is little in these [conventional asset pricing models] that allows for illiquidity. Consequently, the required rate of return will be the same for liquid and illiquid assets with similar market risk exposure.").

[33] *See* David Tabak, *A CAPM-Based Approach to Calculating Illiquidity Discounts,* NERA (Nov. 11, 2002), 17–19, https://www.nera.com/content/dam/nera/publications/archive1/5657.pdf; Juana Alonso Cañadas and Alfonso A. Rojo Ramirez, *The Discount Rate in Valuing Privately Held Companies*, 30 BUS. VALUATION. REV. 1, 1 (2011); Comment, *supra* note 2, at 83; *see also* Z. CHRISTOPHER MERCER & TRAVIS W. HARMS, BUS. VALUATION: AN INTEGRATED THEORY 327–47 (3d ed. 2021) (developing the Quantitative Marketability Discount Model (QMDM)).

[34] *See* Tabak, *supra* note 33, at 17–20.

3. Empirical Models

Finally, the empirical model approach takes the present value of all future transaction costs and subtracts it from the marketable value of the asset. This comes down to valuing the business as a liquid asset and then applying a marketability discount.[35] Typically, the marketability discount estimate is based on empirical studies, which include: (i) restricted stock studies that analyze the price differential between restricted shares and their freely traded counterparts;[36] (ii) the IPO studies that express the DLOM as the percentage difference between the price at which a private company goes public and the private market price of the firm prior to the IPO;[37] and (iii) the valuation multiplier studies that compare acquisition values of private and public companies based on the price multiples of various financial parameters.[38]

Below, Table 1 presents an overview of the various models, including their measurement rationale and indicative DLOM range.

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[35] Bajaj et al., *supra* note 21, at 90 (referring to the empirical model and "[t]he usual valuation methodologies, which utilize cash flows or market transactions, [and] do not explicitly account for the marketability of an asset. Hence, in order to value an asset that is not marketable, the usual approach is to value the asset as if it were marketable, then apply a marketability discount to this estimated value.").

[36] *See* discussion *infra* Section II.B.1 (explaining that restricted stock studies are particularly illustrative for the central theme of this article and are therefore more extensively discussed below).

[37] Bajaj et al., *supra* note 21, at 93. The IPO approach has been criticized for two main reasons. First, this approach suffers from selection bias; only successful IPO companies are included in the sample. Second, it is not always possible to verify the arm's length character of the pre-IPO transactions, Bajaj et al., *supra* note 21, at 94–96.

[38] Experts have argued against the valuation multipliers approach because it leads to a wide range of varying outcomes depending on the financial parameters considered (e.g., price to earnings, price to book, etc.). Also, it cannot be overlooked that the transaction values for both public and private firms may include synergistic considerations.

Table 1.

Model	Method	Measurement rationale	DLOM (indicative)
Financial	1. Put option methods	A put option represents the value of a right to sell a stock. This type of model measures the DLOM by dividing the put option value by the market value of the stock.	25–50%
	2. DCF based models	The illiquidity discount is included in the DCF model, and the implied DLOM is calculated based on the resulting value.	20–50%
Empirical	3. Pre-IPO stock studies	The IPO stock price is compared with the stock price in a private transaction prior to the IPO.	40–50%
	4. Restricted stock studies	A publicly traded entity can issue non-trading stocks in a private placement. Prices of the liquid and restricted stocks are compared.	10–20%
	5. Valuation multipliers	Private acquisitions are matched with public peers, and valuation multipliers are used to compute the valuation discount.	15–30%

B. The Effect of Liquidity on the DLOM

As indicated above, the various approaches and wide-ranging discount outcomes illustrate the difficulty of valuing a private company and determining the private company discount. Customarily, scholars and valuation experts rely on industry practice and established rules of thumb, or they seek out the studies and data that support their arguments.[39]

Historically, scholars and valuation experts understood the value gap between public and private companies as compensation for lack of liquidity, as indicated by its name: discount for lack of *marketability*.[40] However, successive studies have found that the discount also accounts for other distinguishing factors between private and public companies.[41] In that sense, it is probably more correct to refer to a private company discount and to see the restricted liquidity as one component of the price differential between public and private companies. Yet, surprisingly, few

[39] *See* Tabak, *supra* note 33, at 17 (noting that the "final haircut for illiquidity is generally quite subjective."); *see also* Chipalkatti et al., *supra* note 3, at 57 (observing that "the calculation of the DLOM is a subjective process involving many assumptions [that] often degenerates to an ad hoc reduction of a certain percentage from the estimated 'marketable' value of the assets without any economic reasoning.").

[40] *See* Bajaj, et al., *supra* note 21.

[41] Damodaran, *supra* note 22, at 31.

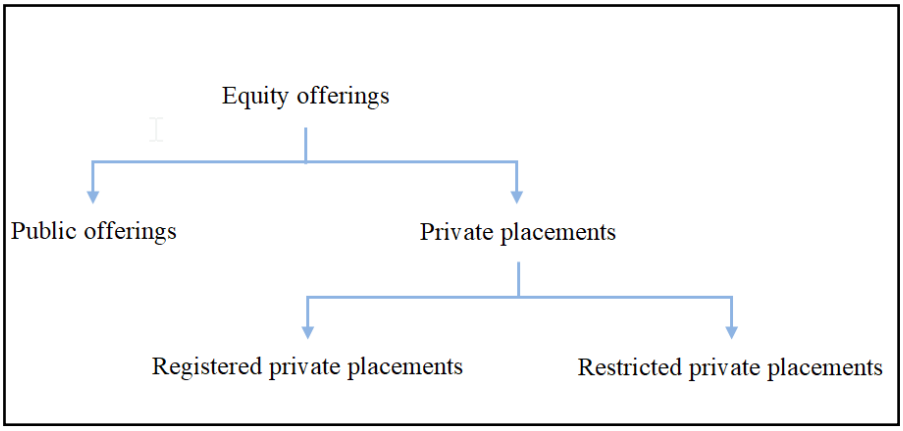
studies examine the separate effect of liquidity (or transfer restrictions) on the valuation discount. We present these studies below:

1. Restricted Stock Studies

A company seeking to refinance or raise capital for expansion can either sell shares on the open market or organize a private placement. A private placement is a sale of stock to pre-selected investors and institutions rather than to the public.[42]

The private placement can concern either restricted stock or registered stock (see Table 2 below). Restricted stock (or letter stock) refers to stock of a publicly traded company that is restricted from trading under Rule 144 of the Securities Act of 1933.[43] Because of the trading restriction, some authors believe that the difference between the price at which restricted stock is sold and the price of the freely traded stock can serve as a proxy for the DLOM.[44]

Table 2.



[42] Damodaran, supra note 22, at 28.

[43] *See* 17 C.F.R. § 230.144(a)(3) (2022) (explaining the Securities Act of 1933 and defining restricted securities).

[44] *See, e.g.*, Milton Gelman, *An Economist-Financial Analyst's Approach to Valuing Stock of a Closely Held Company*, 36 J. TAXATION 353, 354 (1972); Robert E. Moroney, *Most Courts Overvalue Closely Held Stocks*, 51 TAXES - TAX MAG. 144, 148–50, 154–55 (1973); J. Michael Maher, *Discounts for Lack of Marketability for Closely Held Business Interests*, 54 TAXES - TAX MAG. 562, 571 (1976); Robert R. Trout, *Estimation of the Discount Associated with the Transfer of Restricted Securities*, 55 TAXES - TAX MAG. 381, 383 (1977).

Early restricted stock studies conducted in the 1970s found that the average private placement discount of restricted stock fell between 30% and 40%.[45] Then, in 1990, the SEC modified Rule 144 and allowed qualified institutional investors to trade unregistered securities among themselves without filing registration statements.[46] Applying the amended rule to the restricted stock studies, experts found that the restricted stock became more liquid while concurrently reducing the private placement discount to 25% and below.[47] The standard practice in these earlier studies involved limiting the sample to restricted stock only on the assumption that the regulatory restriction single-handedly caused the discount.[48] However, four pivotal studies overturned this interpretation.[49]

William Silber conducted the first of these four studies.[50] In 1991, Silber examined the private placement discounts on restricted stocks, concluding that while liquidity does have a significant impact, the discount also depends on other factors. These factors include, but are not limited to: (1) company revenue; (2) profitability; (3) the fraction of shares placed in the company; and (4) the company's relationship with the investor(s).[51] Ultimately, Silber demonstrated that the regulatory trading restriction was not the only cause of the private placement discount.[52]

Then in 1993, Michael Hertzel and Richard Smith's study compared the private placement of restricted, unregistered stock to the private placement of registered shares.[53] They found that the discount for restricted shares was only 13.5% greater than that of registered shares.[54] This is a

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[45] These early studies include notably: Gelman, *supra* note 44, at 354 (finding both the average and median DLOM was 33%); Moroney, *supra* note 44, at 148–50, 154–55; Maher, *supra* note 44**,** at 571 (finding the average DLOM was about 35% across multiple studies); Trout, *supra* note 44, at 383 (finding an average DLOM of 33.45%).

[46] Robert Reilly & Aaron Rotkowski, *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*, 61 TAX LAW. 241, 257–58 (2007) (explaining that in 1990, the rule was amended to allow qualified institutional investors to trade unregistered securities among themselves. The U.S. Securities and Exchange Commission (SEC) reduced the mandatory holding period for restricted stock from two years to one-year, effective April 29, 1997).

[47] *Id.* at 258 ("Rather, the liquidity of restricted securities has increased. As it became easier to find a buyer for restricted securities after 1990, the average restricted stock price discount decreased.").

[48] *See* Comment, *supra* note 10, at 91.

[49] *See* discussion *infra* Section II.B (1) – (2).

[50] William L. Silber, *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*, 47 FIN. ANALYSTS J. 60, 60–64 (1991).

[51] *Id.*

[52] *Id.* at 63–64.

[53] Michael Hertzel & Richard L. Smith, *Market Discounts and Shareholder Gains for Placing Equity Privately*, 48 J. FIN. 459, 477–80 (1993).

[54] *Id.* at 480.

more reliable estimate of the marketability discount because it directly compares the private placement of restricted stock (subject to transfer restrictions) and the private placement of registered stock that can be freely traded.[55] Nevertheless, Hertzel and Smith cautiously concluded that "it [was] unlikely that a pure illiquidity effect [could] explain the magnitude of the discounts [they found]."[56]

The third and fourth studies were conducted by Mukesh Bajaj et al. in 2001 and Robert Comment in 2012.[57] Bajaj et al. found a 14.09% difference between the discount for unregistered and registered private placements.[58] They introduced three control variables in a multivariate regression: (1) the percentage of shares issued; (2) an indicator of the firm's financial health; and (3) the standard deviation of the firm's returns. They determined that liquidity was not the only variable impacting the marketability discount, which they concluded was only 7.23%.[59] Likewise, Comment's study utilized the same method and examined fifteen explanatory variables that reflect solvency, buyer skepticism, the potential for dilution, arm's length character, and marketability.[60] He concluded that the discount, which was tied directly to the regulatory restriction, was at most 5.2%.[61]

2. Other Studies

In addition to the restricted stock studies, the limited effect of marketability on the DLOM has been suggested in other types of studies as well.[62] For example, Comment's 2012 study employs a DCF model, concluding that an increased discount rate compensates for the private company's size as compared to public companies rather than for restricted liquidity.[63] Comment's study suggests that firms may only justify a marketability discount of a few percentages.[64]

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[55] *See id.*
[56] *Id.*
[57] *See* Bajaj et al., *supra* note 21; *see also* Comment, *supra* note 10.
[58] *See* Bajaj et al., *supra* note 21, at 107.
[59] Bajaj et al., *supra* note 21, at 113–14.
[60] *See* Comment, *supra* note 10, at 87–89.
[61] Comment, *supra* note 10, at 89.
[62] *See generally* Comment, *supra* note 10.
[63] *See generally* Comment, *supra* note 10.
[64] *See generally* Comment, *supra* note 10, at 90–91. Comment identified the size premium that is included in the discount rate based on fairness opinions and shows WACC differences of 8.8% between the smallest and the largest companies in his sample; *see* Comment, *supra* note 10, at 81.

In their option-based model, Barenbaum et al. suggest that the proceeds of a hedging strategy employing an at-the-money equity collar, along with the interest cost of borrowing funds to monetize the underlying asset, captures the cost of providing liquidity to a non-traded asset.[65] They show that the net cost of such a strategy is significantly below the cost of a put option and comparable to the percentages advanced by Bajaj et al. and Comment.[66]

Other authors have proposed that the value of liquidity is simply equal to the cost of selling or going public.[67] The estimates advanced in these studies range broadly from 2% to 12%.[68]

C. Current Consensus and Contribution

The above-referenced studies demonstrate that transfer restrictions (liquidity) have a notable impact on the DLOM, but that effect may be limited to a few percentage points. In addition to these studies, experts also point out that some private placements and transactions involve a premium—a fact that cuts against the traditional view that marketability alone explains the value difference between public and private companies.[69]

Several other explanations have been advanced, especially in the private placement studies.[70] For example, the monitoring and ownership concentration theory holds that a private placement leads to a concentration of ownership and that the private placement discount is

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[65] *See* Barenbaum et al., *supra* note 29, at 66.

[66] *See* Barenbaum et al., *supra* note 29, at 68–69.

[67] *See* discussion and sources cited *infra* note 68.

[68] The cost of going public has been estimated between 2% and 10%, depending on the size of the transaction with an average of 5.36% for underwritten IPOs. *See* Reilly & Rotkowski, *supra* note 42. Additionally, Comment examines 552 underwritten IPOs between 2004 and 2009, arriving at a similar cost of about 5.8%. *See* Comment, *supra* note 2. However, these estimates typically exclude the 7% underwriter price discount. *See*, *e.g.*, Hsuan-Chi Chen & Jay R. Ritter, *The Seven Percent Solution*, 55 J. FIN. 1105, 1105 (2000). Further, Bruner and Palacios mention the indirect costs (underpricing), which they estimate to be 15%. *See* Robert Bruner & Miguel Palacios, Valuing Control and Marketability, 1–40 (May 28, 2004) (unpublished working paper) (on file with the Batten Inst., Univ. Va. Darden Sch. Bus.). Also, certain authors calculate the DLOM based on selling expenses; in other words, they estimate the discount based on the hypothetical costs to sell, which would reduce the proceeds if the company were sold. For example, Anderson and Long refer to commissions ranging from 3% to 12%, depending on the size of the firm. *See* Anthony J. Anderson & Michael S. Long, *Evidence on Lack of Liquidity for Small Public Firms*, 7 J. BUS. VALUATION & ECON. LOSS ANALYSIS 1 (2012).

[69] *See* Karen H. Wruck & YiLin Wu, *Relationships, Corporate Governance, and Performance: Evidence from Private Placements of Common Stock*, 15 J. CORP. FIN. 1, 1 (2009).

[70] *See* sources cited *infra* note 71.

compensation for monitoring by the new owners.[71] Another school of thought explains the discount as a compensation for information acquisition costs by the investors.[72] Comparatively, the management entrenchment theory holds that management organizes the private placements in such a manner that they can solidify their control over the firm.[73] Therefore, they prefer passive investors, and the discount can be explained as a "reward" for the investors' passivity.[74] Yet other scholars have attributed the discount to the issuing firm's financial situation; thus, if investors suspect that the firm is experiencing financial difficulties, they may ask for a discount on the share price.[75]

In summary, the current view holds that, in addition to marketability, various factors impact the total valuation discount. It is important to stress that most approaches and models are based on, or derived from, public company information. As such, there is a lack of direct empirical evidence for valuation discounts on privately held firms because of the non-availability of information.[76] Our dataset is thus a welcome addition for two reasons: (1) because it is based entirely on private company information; and (2) because our data source provides a wide range of arguments and explanatory variables for which we can test their impact on the DLOM.

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[71] Karen H. Wruck & YiLin Wu, *supra* note 69; *see also* Bajaj et al., *supra* note 21, at 106; Karen H. Wruck, *Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financing*, 23 J. FIN. ECON. 3, 4–5 (1989).

[72] In certain industries, there is a high level of information asymmetry. The new investors face important due diligence (information acquisition) costs, and the discount compensates these efforts. S*ee*, *e.g.,* Hertzel & Smith, *supra* note 53, at 480–81; Srinivasan Krishnamurthy et al., *Does Investor Identity Matter in Equity Issues? Evidence from Private Placements*, 14 J. FIN. INTERMEDIATION 210 (2005); Rahsan B. Inget, *Private Equity Placements and Illiquidity Discount*, SSRN (Oct. 2009), https://ssrn.com/abstract=1583562.

[73] Michael Barclay et al., *Private Placements and Managerial Entrenchment*, 13 J. CORP. FIN. 461, 461 (2007).

[74] *See id.* at 462; *see also* Krishnamurthy et al., *supra* note 72, at 211; Wruck & Wu, *supra* note 69; YiLin Wu, *The Choice of Equity-Selling Mechanisms*, 74 J. FIN. ECON. 93, 98 (2004).

[75] This view holds that the discount compensates private investors for their willingness to provide capital when public funding would be difficult to secure. *See, e.g.*, Shin-Herng Michelle Chu et al., *Comparing the Characteristics and Performance of Private Equity Offering Firms with Seasoned Equity Offering Firms*, 1 J. ECON. MGMT. 57, 58–59 (2005); *see also* Hertzel & Smith, *supra* note 53.

[76] Ashish Kumar Garg & Kundan Kumar, *Option Pricing Models of Private Equity Valuation: A Comparative Analysis*, 20 IUP J. APPLIED FIN. 28, 29 (2014).

III. DATASET AND TAXONOMY

A. Data Source

Due to the shortcomings of traditional methods, we explored alternative sources of information. Specifically, we turned to U.S. court decisions on gift and estate taxes, spanning from January 1990 to June 2021, in which the court determined the fair market value of private companies.[77] For the purposes of our research and analysis, we focused on gift and estate tax cases for several advantageous reasons: (1) they apply an unequivocal fair market standard value;[78] (2) because of their focus on the fair market value, estate, and gift tax cases may apply a DLOM as well as a minority discount;[79] and (3) estate and gift tax cases are subject to standardized federal law, thereby avoiding the complexity of conflicting state law.

Based on a keyword search in LexisNexis, we identified 270 total cases.[80] To narrow the list, we conducted a thorough prescreening, rejecting cases that did not contain an unambiguous DLOM decision for a private company.[81]

After examining and omitting the cases that did not meet our criteria, eighty cases remained. From these cases, we pulled 137 useful

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[77] "Fair market value is defined as the price at which property would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or sell and both having reasonable knowledge of relevant facts." *See Estate of Marmaduke*, 1999 WL 818788, at *2.

[78] By contrast, appraisal cases typically use the "fair value" standard. "Fair value" is an ambiguous term that can be interpreted at will. In determining fair value in the context of appraisal rights, most (but not all) jurisdictions regard shareholder level minority and liquidity discounts as prohibited as a matter of law. *See* Stephen J. Leacock, *Lack of Marketability and Minority Discounts in Valuing Close Corporation Stock: Elusiveness and Judicial Synchrony in Pursuit of Equitable Consensus*, WM. & MARY BUS. L. REV. 683, 722–23 (2016). They do this notably on the premise that the majority shareholder would otherwise reap a windfall at the expense of the minority. *See* Gilbert Matthews, *Statutory Fair Value in Dissenting Shareholder Cases: Part I*, 36 BUS. VALUATION REV. 15, 17 (2017).

[79] The fair market value test does not preclude the application of either lack of control or lack of marketability discounts. *See* Leacock, *supra* note 78, at 730.

[80] We have identified 270 potential cases based on the keywords "discount" and "lack of marketability" with results narrowed down by practice area (tax law), jurisdiction (federal), and timeline (cases decided after January 1, 1990).

[81] We also dismissed cases for the following reasons: if the court did not provide a clear or final decision on the DLOM; if the case dealt with discounts for assets (e.g., fractional interests in real estate) and not for a company; if the case did not provide enough detail; if the court was valuing a public company; or if the case was overruled.

observations.[82] We assigned two independent reviewers per case to examine, compare, and consolidate their findings. For each of the 137 observations, the reviewers recorded the DLOM as well as a list of potential explanatory variables (determinants that have been suggested in the extant literature and a number of other variables that we could identify across our dataset). Overall, the rigid case selection and inter-rater reliability guaranteed a qualitative dataset. An overview of the court cases and the number of observations in each of these cases can be found in Exhibit A.[83]

B. The Value of Case Analysis

The impact of transfer restrictions and liquidity on the DLOM is a recurrent theme in U.S. tax valuation cases where the court examines the fair market value of stock holdings in private companies.[84] For example, in the landmark case of *Mandelbaum v. Commissioner*, the court lists transfer restrictions and the redemption policy as two of ten fundamental elements of value explaining the discount for lack of marketability.[85] Similarly, in *Estate of McCormick v. Commissioner*, the court ruled that "restrictions on transferability" are one of the traditional factors that impact marketability.[86] These cases provide details normally unavailable about private companies, showcasing novel insights into the court's reasoning as they relate to a DLOM decision. This allows us to assess what

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[82] From the eighty cases, we documented 137 observations because some cases decided on the valuation of more than one company or on multiple transactions in the same company. *See infra* Exhibit A.

[83] *See* list of cases *infra* Exhibit A.

[84] *See supra* text accompanying notes 79–81.

[85] The *Mandelbaum* decision lists the following nonexclusive factors: "(1) [t]he value of the subject corporation's privately traded securities vis-a-vis its publicly traded securities (or, if the subject corporation does not have stock that is traded both publicly and privately, the cost of a similar corporation's public and private stock); (2) an analysis of the subject corporation's financial statements; (3) the corporation's dividend-paying capacity, its history of paying dividends, and the amount of its prior dividends; (4) the nature of the corporation, its history, its position in the industry, and its economic outlook; (5) the corporation's management; (6) the degree of control transferred with the block of stock to be valued; (7) any restriction on the transferability of the corporation's stock; (8) the period of time for which an investor must hold the subject stock to realize a sufficient profit; (9) the corporation's redemption policy; and (10) the cost of effectuating a public offering of the stock to be valued, e.g., legal, accounting, and underwriting fees." Mandelbaum v. Comm'r, No. 20517-92, 1995 WL 350881, at *36–37 (T.C. Jun. 12, 1995).

[86] The court also mentions the size of the interest being valued and the risks inherent in the business conducted by the entity. Estate of McCormick v. Comm'r, No. 18417-91, 1995 WL 462283, at *21 (T.C. Aug. 7, 1995).

roles transfer restrictions and liquidity play as two of many attributes to the marketability discount.[87]

C. Taxonomy

An asset is not solely characterized as either marketable or nonmarketable. Rather, there are varying degrees of marketability and transfer restrictions.[88] While there are various ways to formulate and classify transfer restrictions, Stephen Leacock proposes the following classification:[89]

- Absolute prohibitions against transfer
- Consent restraints
- First refusal rights
- Options to purchase by the corporation or other shareholders
- Mandatory buy-sell agreements [90]

For our purposes, we based our categorization on an inductive analysis of gift and estate tax courts cases. Not only did our categorization cover the items proposed by Leacock, but it also allowed us to capture the combined effect of liquidity and transfer restriction arrangements. Indeed, certain arrangements can be qualified as both transfer restriction and a source of liquidity. For example, an approval clause can be designed in such a way that its exercise leads to the activation of a put option for the

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[87] Estate of Simpson v. Comm'r, No. 16581-91, 1994 WL 184404, at *15 (T.C. May 11, 1994) (citing that "[t]he valuation of [a] … stock is a question of fact, with respect to which the court must weigh all the evidence and draw appropriate inferences. Hamm v. Comm'r, 325 F.2d 934, 938 (8th Cir. 1963). In the absence of an open market for the stock, it is proper to consider all the facts and circumstances related to the corporation in determining the fair market value of its stock. O'Malley v. Ames, 197 F.2d 256, 257–58 (8th Cir. 1952). In valuing stock, [the court] is not bound by the opinions of the expert witnesses. Instead, [the court] may reach a decision on the basis of its own analysis of all of the evidence in the record." Silverman v. Comm'r, 538 F.2d 927, 933 (2d Cir. 1976)).

[88] *See* PRATT & NICULITA, *supra* note 1, at 446.

[89] Stephen J. Leacock, *The Anatomy of Valuing Stock in Closely Held Corporations: Pursuing the Phantom of Objectivity into the New Millennium*, 2001 COLUM. BUS. L. REV. 161, 170 (2001); *see* Hertzel & Smith, *supra* note 53 (discussing the formulation of categories of restrictive agreements).

[90] Mandatory buy-sell agreements typically require the estate of the decedent to sell and the corporation to buy the decedent's shares at a fixed price; or the agreements are based upon a predetermined formula. *See* Leacock, *supra* note 89.

shareholder. As such, this combination of effects must be considered and tested in parallel.[91]

Accordingly, we have catalogued the restrictions and liquidity arrangements separately, and we have distinguished three classes within each variable.

1. Classes of Transfer Restrictions

Class A: Under this classification, the court cases do not identify any transfer restrictions on shares. In essence, the transfer restrictions are "absent" (or at least not advanced as an argument in the court decision).[92]

Class B: Under this classification, the court cases discuss the right of first refusal rights and equivalent arrangements that can delay a proposed transfer (or payment) for up to sixty days without impact on the price (i.e., classic right of first refusal arrangements based on the price offered by the prospective purchaser).[93]

Class C: Under this classification, restrictions include right of first refusal rights at a formula price and arrangements that can delay the transfer or payment for more than sixty days. Also, within Class C, we include approval (veto) rights which gives the board, shareholder, or a third party the right to simply block a proposed transfer.

2. Classes of Liquidity

Class A: This liquidity classification covers the cases in which we did not find any indication of liquidity. Shareholders need to find their own way out of the company by finding a purchaser, agreeing on a price, and/or

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[91] In the landmark *Mandelbaum* case, the court not only mentions transfer restrictions as a possible determinant of the DLOM, but also the corporation's redemption policy. *See Mandelbaum*, 1995 WL 350881, at *36–37.

[92] Where the cases discuss an obligation to inform the company about a proposed transfer, we do not consider this a restriction per se.

[93] We note that the effect of a right of first refusal (ROFR) is more than a technical delay in the transfer. Such a right can have an adverse effect on the willingness of a potential acquirer to consider bidding. *See* Matthews, *supra* note 13, at 3. On the other hand, as early as 1994 courts have observed that "a right of first refusal will have little, if any effect on fair market value and clearly will have a less drastic effect than fixed price restrictions." *See* Estate of Luton v. Comm'r, No. 23339-91, 1994 WL 589560, at *16 (T.C. Oct. 27, 1994). The courts thus make a distinction between a ROFR that is based on a formula (fixed) price and a ROFR based on the price offered by the prospective purchaser. The reason is that a formula or fixed price puts the selling shareholder in a position where he may receive less than the price for which he wanted to sell. For this reason, we have included these severe arrangements in a separate category C (along with arrangements that can delay the transfer and/or payment for more than 60 days).

where applicable, respecting the restrictions and procedures imposed by the company.

Class B: This classification includes situations where the company is not systematically organizing or sponsoring the liquidity; however, the decision points to instances of occasional redemption by the company or of a confirmed interest (by either an existing shareholder or a third party) to acquire a stake that is proposed for sale.

Class C: This classification contains instances of organized liquidity. This can take the form of a systematic and organized redemption policy or of contractual rights (i.e., a put option) that provide an exit solution for shareholders that want to sell their stake.

Based on the current literature, we hypothesize that the transfer restrictions, depending on their severity, increase the DLOM; and conversely, depending on its availability to shareholders, liquidity decreases the DLOM.

IV. METHODOLOGY

A. Impact of Transfer Restrictions

We used a regression analysis to verify whether the classes of transfer restrictions and liquidity had an impact on the discount for lack of marketability.[94] A regression analysis is a recognized statistical method to estimate the relationship between a dependent variable and one or more independent variables. In our study, we applied the DLOM, as decided by the court, as our dependent variable. For the independent variables, we applied the classifications of transfer restrictions and liquidity and several control variables, as discussed below.[95]

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[94] We use a linear regression analysis in this article. We note that since our output variable (DLOM) is a percentage that can take any value from zero (0%) to one (100%), a theoretically more correct approach would be to apply a fractional regression analysis. *See* Michael Clark, *Fractional Regression*, STATISTICS, SCIENCE, DATA (Aug. 20, 2019), https://m-clark.github.io/posts/2019-08-20-fractional-regression/. However, since the observed DLOM values in our dataset steer away from the theoretical boundaries, a linear regression will lead to approximately the same result. We prefer to report the linear regression results because they are easier to interpret.

[95] The output results indicate not only the estimated size effect of the dependent variables on the DLOM but give us two important additional indicators: the p-value indicates the probability that the observed effect is due to random sampling error. Convention in social sciences such as law or economics dictate that a p-value below 5% indicates a significant finding. The other relevant indicator is the R-Squared (R^2 or the coefficient of determination). This is a statistical measure in a regression model that determines the proportion of variance in the dependent variable that can be explained by the independent variable. In other words, R-squared is the explanatory value: a measure that shows how well the data fit the regression model. The

For the purposes of our analysis, we present first a standalone regression result for the Class B and C transfer restrictions and the Class B and C types of liquidity.[96] The first regression result is presented in Table 3 and shows the impact of transfer restrictions and company-sponsored liquidity on a company's DLOM (the intercept in the table). As shown in Table 3, the results indicate that Class B and Class C transfer restrictions increase the DLOM. However, the p-value for the Class B restrictions is well above the accepted threshold of 5%; therefore, only the impact of Class C restrictions is conclusive. The same applies to the Class B and Class C types of liquidity. Both types of liquidity decrease the DLOM, but the p-value of the Class B type of liquidity is too high for the results to be conclusive. The p-value for the Class C of liquidity borders on the 5% threshold.

Table 3.

Variable	Estimate	Std. Error	t-statistic	p-value
Intercept	22.192	1.161	19.120	.000
B restrictions	.401	2.014	.199	.842
C restrictions	4.789	1.733	2.763	.007
B liquidity	-1.375	2.175	-.632	.528
C liquidity	-6.110	3.100	-1.971	.051

The fact that only Class C transfer restrictions have a significant effect on the DLOM may seem surprising. Nevertheless, the landmark *Mandelbaum* case already hinted at the different effects of Class B and Class C restrictions.[97] As stated in *Mandelbaum*, a classic right of first refusal (based on the third party offered price) "merely governs the order in which prospective buyers must stand in line to buy the stock."[98] Furthermore, the court reasoned that because the right benefits and protects shareholders by giving them the right to purchase if a fellow shareholder wants to sell, "the depressant effect (if any) on the value of

adjusted R-squared is a modified version of R-squared that has been adjusted for the number of independent variables in the model.

[96] The Class A transfer restrictions and liquidity are the default case where there are no transfer restrictions and no company-sponsored liquidity. This situation is reflected in the intercept.

[97] *See Mandelbaum*, 1995 WL 350881, at *36–37.

[98] *Id.* at *14.

privately held stock subject to a right of first refusal is not necessarily substantial."[99]

B. Impact of Control Variables

After demonstrating the stand-alone impact of hard transfer restrictions and organized liquidity on the DLOM (and thus on a company's value), we controlled for effects that are statistically attributable to other explanatory variables.

In our selection of control variables, we were guided by the Bajaj et al. and Comment studies in which they determined the isolated effect of regulatory trading restrictions on the DLOM.[100] Starting with their explanatory variables, and guided by extant literature and our dataset, we tailored the final set of control variables to include the following:[101]

Size of the company: Here we added the total undiscounted equity value of the company at the valuation date (in million USD) to the regression.[102] The traditional view holds that company size negatively correlates to risk and positively correlates to liquidity.[103] In other words, the larger the company, the lower the risk, and the lower the risk, the lower the DLOM.

Size of the interest: The DLOM might depend on the size of the interest being valued (i.e., the number of shares expressed as a percentage of the total number of shares outstanding). An important block of shares is associated with a higher DLOM. This fact relies on the logic that the discount compensates the shareholder for their efforts to monitor the firm.[104]

Profitability: In theory, a financially healthy firm will have a lower DLOM. Hence, we expect that our indicator variable for profitability will negatively impact the DLOM.[105] This indicator denotes whether the company was profitable in the year preceding the valuation date.

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[99] *Id.*

[100] *See* Bajaj et al., *supra* note 21, at 114; *see also* Comment, *supra* note 10, at 91.

[101] Bajaj et al. and Comment employed restricted stock studies that include by design only companies for which there is an established trading forum. *See* Bajaj et al., *supra* note 21, at. 112–13; *see also* Comment, *supra* note 10, at 86–87.

[102] Because we compared company sizes over a long period, we adjusted all values based on the consumer price index (CPI) published by the World Bank.

[103] *See* Comment, *supra* note 10, at 90.

[104] *See supra* Section II.C. (aligning with the ownership and monitoring theory referenced).

[105] Indicator variables (or dummy variables) are constructs that equal one when a given condition prevails and zero otherwise.]

Information: When companies provide verified, reliable information to prospective shareholders, they reduce information asymmetry, therefore decreasing prospective shareholder risk. This should lead to a lower DLOM. To use an objective measure for the information variable, we have introduced a dummy variable indicating whether a company provides externally audited accounts.

Standard deviation of the firm's returns: This variable measures the business's risk and uncertainty. Because publicly traded share prices are unavailable for private firms, we resorted to a proxy: the spread between the parties' final valuations submitted to the court. Specifically, our measure is the difference between the two valuations divided by the average valuation.[106]

Industry: About half of the observations in our dataset concern companies with two-digit SIC codes within the sixty to sixty-seven range (Division H).[107] The other observations are scattered across the other SIC division codes.[108] In our study, we introduced an indicator variable for division H companies.

DLOC: We used the discount for lack of control (DLOC) as an additional control variable.[109] Most courts follow the economic theory that the DLOC is distinguishable from the DLOM.[110] However, some courts are ambiguous in their analysis, suggesting that there might be an overlap.[111]

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[106] This can be presented as the following formula $(hi - lo)/((hi + lo)/2)$ whereby "hi" and "lo" denote the highest and lowest valuation presented to the court. The spread is expressed as a percentage and can theoretically take any value between 0% and 200%.

[107] The two-digit SIC codes are catalogued within Division H and include establishments operating primarily in the fields of finance, insurance, and real estate. *See 2-Digit SIC (Standard Industrial Classification) Codes*, N.C. STATE UNIV.: MCKIMMON CENTER FOR EXTENSION & CONTINUING EDUC. (June 16, 2021, 10:49 AM), https://mckimmoncenter.ncsu.edu/2digitsiccodes/.

[108] *Id.* We present an overview with the observations for the different industry divisions in Exhibit B.

[109] From an econometrical point of view, the DLOC is to be considered as a predetermined variable. In *Estate of Magnin v. Commissioner*, the Court decided that "[i]n order to ensure accuracy, the minority interest discount should be applied first and then the marketability and liquidity discount should be applied to this figure," No. 24883-92, 2001 WL 117645, at *18 (T.C. Feb. 12, 2001). Our dataset confirms that the courts systematically apply a DLOC (where appropriate) before deciding on the DLOM.

[110] *See* list of cases *infra* Exhibit A.

[111] This potential overlap was suggested as early as 1982. *See, e.g.*, Estate of Andrews v. Comm'r, 79 T.C. 938, 953 (1982) (stating that "[t]he minority shareholder discount is designed to reflect the decreased value of shares that do not convey control of a closely held corporation. The lack of marketability discount, on the other hand, is designed to reflect the fact that there is no ready market for shares in a closely held corporation. *Although there may be some overlap between these two discounts in that lack of control may reduce marketability,* it should be borne in mind that even controlling shares in a nonpublic corporation suffer from lack of marketability

Table 4.

Variable	Estimate	Std. Error	t-statistic	p-value
Intercept	22.262	3.119	7.137	.000
C restrictions	4.670	1.400	3.336	.001
C liquidity	-10.418	2.682	-3.884	.000
Company size	.004	.002	2.453	.016
Size of the interest	.009	.026	.367	.714
Profitability	1.106	2.220	.498	.619
Audit	2.038	1.916	1.064	.290
Spread	-.032	.021	-1.475	.143
Industry H	-5.730	1.371	-4.181	.000
DLOC	.230	.080	2.880	.005
R-squared	.383			
Adjusted R-squared	.331			

C. Discussion

The regression results that include control variables allowed us to validate our conclusions. We found that Class C contractual transfer restrictions increase the DLOM by approximately 4.7%, negatively impacting a company's value. Conversely, the introduction of organized micro-liquidity decreases the DLOM by more than 10%.

Oftentimes, a company's charter and shareholder agreements contain boilerplate transfer restrictions or liquidity provisions lacking consideration. However, as indicated by the figures above, the importance of these terms should not be overlooked. Rather, their enormous impact on a company's value should incite executives and shareholders to consider the necessity and construction of transfer restrictions as well as the inclusion of redemption policies or similar arrangements.

Likewise, we found that other certain variables also impact the DLOM. For instance, Division H entities—holding, finance, and real estate companies—have a lower DLOM; we suspect that they may be predominantly holding companies for which appropriate discounts have

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because of the absence of a ready private placement market and the fact that flotation costs would have to be incurred if the corporation were to publicly offer its stock.") (emphasis added).

already been applied in the valuation of the underlying assets. Also notable, the size of the company appears statistically significant, but its effect on the DLOM is small. Furthermore, the size differences between the private companies in our sample do not appear to be important. Instead, we assume that the size difference between private and public companies plays a more significant role with regards to the DLOM.[112] Finally, the interaction between control and marketability stands out. The coefficients seem to indicate that less control (as reflected in a higher DLOC) leads to a higher DLOM. Unfortunately, the relation between these two supposedly separate and different discounts has received scant attention in the literature.

In closing, we highlight the significance of our model's R-squared. In a model that aims to explain human behavior (in our case, legal decisions) within a complex environment, an adjusted R-squared of 33.1% is a high value.

V. CONCLUSION

The size of the DLOM and the effect of liquidity on this discount remain hotly debated topics. The uncertainty is a welcome argument in appraisal, gift tax, inheritance, and divorce matters.[113] Richard Sansing notes that, in the context of disputes, "the enormous differences in valuation discounts arise in part because the justification for valuation discounts has been based on casual heuristics rather than formal economic reasoning."[114] Comment acknowledged that high discounts reflect redundant discounting.[115] However, he also conceded that this has become a standard practice in business valuation, and as a result, is accepted by many judges.[116]

Our dataset suggests that when valuing private companies, the courts apply an average DLOM of 23.75% based on the total discounts identified in various empirical studies. Nevertheless, our study and subsequent results also prove that contractual transfer restrictions and the organization of liquidity have a significant effect on the DLOM decided by the courts and, thus, on a company's value. Depending on their nature, transfer restrictions decrease a company's value by approximately 4.7%;

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[112] The average size of the companies (equity value) in our sample is $120.3 million.

[113] *See* Comment, *supra* note 10, at 80.

[114] *See* Comment, *supra* note 10, at 80.

[115] *See* Comment, *supra* note 10, at 80.

[116] Comment's 2010 study suggests that a large DLOM is likely to just be a discount for lack of size (DLOS) by another name. *See generally* Comment, *supra* note 2.

and organized liquidity may increase the value by more than 10%. Our findings bring to light four important conclusions:

First, not all transfer restrictions are equal. Rather, only the hard Class C transfer restrictions (i.e., rights of first refusal based on a formula price and arrangements that can block or delay a transfer or payment for more than 60 days) have a significant impact.

Second, companies may organize liquidity for the shareholders (most often in the form of a redemption policy). However, only an organized and systematic redemption will have an impact on value. Ad-hoc repurchases to remedy individual situations have no impact.

Third, our court cases method is a novel, empirical approach, confirming the conclusions found in the various studies that endeavored to identify the impact of liquidity on the DLOM. Earlier studies verified a limited impact of regulatory trading restrictions; we find a comparable impact for contractual transfer restrictions.

Fourth, companies may partially control the DLOM, and thereby partially control their value. As demonstrated in our study, companies that opt out of severe transfer restrictions are rewarded with a higher company value, and the same is true for companies that organize a limited but systematic form of liquidity for their shareholders.

To conclude, we would like to discuss the potential weaknesses in our study and suggest areas for future research.

Our results are based on 137 observations. These include all available data points in estate and gift tax cases for the period of January 1990 to June 2021. However, there are many other DLOM decisions not included in our sample, such as appraisal cases and divorce cases. Furthermore, we based our dataset on United States court decisions only. We do not see any obvious reasons why the results would not be valid in other settings, but additional evidence from other jurisdictions would be a welcome addition to our study. The low discount applied on holding companies (Division H companies) that is reflected in our multivariate regression calls for further research. We also believe that the relationship between control and marketability discounts (DLOM and DLOC) warrants more attention.

VI. EXHIBIT A: CASES AND DATA POINTS

#	Date	Case	Obs.	Reporter
1	3-May-2021	Estate of Jackson v. Comm'r	1	T.C. Memo 2021-48 *; 2021 Tax Ct. Memo LEXIS 74 **; 121 T.C.M. (CCH) 1320
2	27-Oct-2020	Lucero v. United States	1	2020 U.S. Dist. LEXIS 199605 *; 2020 WL 6281591
3	10-Jun-2020	Nelson v. Comm'r	2	T.C. Memo 2020-81 *; 2020 Tax Ct. Memo LEXIS 79 **
4	2-Mar-2020	Grieve v. Comm'r	2	T.C. Memo 2020-28 *; 2020 Tax Ct. Memo LEXIS 28 **
5	19-Aug-2019	Estate of Jones v. Comm'r	2	T.C. Memo 2019-101 *; 2019 Tax Ct. Memo LEXIS 108 **
6	26-Mar-2019	Kress v. United States	3	372 F. Supp. 3d 731 *; 2019 U.S. Dist. LEXIS 49850 **; 2019-1 U.S. Tax Cas. (CCH) P60,711
7	24-Oct-2018	Estate of Streightoff v. Comm'r	1	T.C. Memo 2018-178 *; 2018 Tax Ct. Memo LEXIS 179 **; 116 T.C.M. (CCH) 437
8	9-Dec-2015	Redstone v. Comm'r	1	T.C. Memo 2015-237 *; 2015 Tax Ct. Memo LEXIS 242 **; 110 T.C.M. (CCH) 564
9	11-Feb-2014	Estate of Richmond v. Comm'r	1	T.C. Memo 2014-26 *; 2014 Tax Ct. Memo LEXIS 26 **; 107 T.C.M. (CCH) 1135
10	18-Oct-2013	Estate of Tanenblatt v. Comm'r	1	T.C. Memo 2013-263 *; 2013 Tax Ct. Memo LEXIS 273 **
11	8-Apr-2013	Estate of Koons v. Comm'r	1	T.C. Memo 2013-94 *; 2013 Tax Ct. Memo LEXIS 98 **; 105 T.C.M. (CCH) 1567
12	7-Feb-2013	Estate of Kite v. Comm'r	1	T.C. Memo 2013-43 *; 2013 Tax Ct. Memo LEXIS 43 **; 105 T.C.M. (CCH) 1277
13	28-Jun-2011	Estate of Gallagher v. Comm'r	1	2011 Tax Ct. Memo LEXIS 150 *; T.C. Memo 2011-148; 101 T.C.M. (CCH) 1702
14	22-Jun-2011	Estate of Giustina v. Comm'r	1	2011 Tax Ct. Memo LEXIS 141 *; T.C. Memo 2011-141; 101 T.C.M. (CCH) 1676
15	13-May-2010	Pierre v. Comm'r	1	2010 Tax Ct. Memo LEXIS 143 *; T.C. Memo 2010-106; 99 T.C.M. (CCH) 1436
16	2-Oct-2009	Estate of Murphy v. United States	3	2009 U.S. Dist. LEXIS 94923 *; 2009-2 U.S. Tax Cas. (CCH) P60,583; 104 A.F.T.R.2d (RIA) 2009-7703
17	29-Jan-2009	Estate of Marjorie deGreeff Litchfield v. Comm'r	2	2009 Tax Ct. Memo LEXIS 21 *; T.C. Memo 2009-21; 97 T.C.M. (CCH) 1079
18	22-Jul-2008	Bergquist v. Comm'r	2	131 T.C. 8 *; 2008 U.S. Tax Ct. LEXIS 20 **; 131 T.C. No. 2
19	27-May-2008	Holman v. Comm'r	3	130 T.C. 170 *; 2008 U.S. Tax Ct. LEXIS 12 **; 130 T.C. No. 12
20	5-May-2008	Astleford v. Comm'r	3	2008 Tax Ct. Memo LEXIS 129 *; T.C. Memo 2008-128; 95 T.C.M. (CCH) 1497
21	28-Sep-2006	Dallas v. Comm'r	2	2006 Tax Ct. Memo LEXIS 216 *; T.C. Memo 2006-212; 92 T.C.M. (CCH) 313
22	9-May-2006	Huber v. Comm'r	1	2006 Tax Ct. Memo LEXIS 97 *; T.C. Memo 2006-96; 91 T.C.M. (CCH) 1132; RIA TM 56510
23	10-Mar-2006	Temple v. United States	4	423 F. Supp. 2d 605 *; 2006 U.S. Dist. LEXIS 16171 **; 2006-1 U.S. Tax Cas. (CCH) P60,523; 97 A.F.T.R.2d (RIA) 2006-1649
24	11-Oct-2005	Estate of Kelley v. Comm'r	1	2005 Tax Ct. Memo LEXIS 236 *; T.C. Memo 2005-235; 90 T.C.M. (CCH) 369
25	31-May-2005	Estate of Jelke v. Comm'r	1	2005 Tax Ct. Memo LEXIS 128 *; T.C. Memo 2005-131; 89 T.C.M. (CCH) 1397
26	15-Mar-2005	Estate of Bongard v. Comm'r	2	124 T.C. 95 *; 2005 U.S. Tax Ct. LEXIS 8 **; 124 T.C. No. 8
27	26-Jul-2004	Estate of Thompson v. Comm'r	1	499 F.3d 129 *; 2007 U.S. App. LEXIS 20066 **; 2007-2 U.S. Tax Cas. (CCH) P60,546; 100 A.F.T.R.2d (RIA) 2007-5792
28	29-Dec-2003	Estate of Green v. Comm'r	1	2003 Tax Ct. Memo LEXIS 348 *; T.C. Memo 2003-348; 86 T.C.M. (CCH) 758; RIA TM 55384
29	25-Sep-2003	Peracchio v. Comm'r	1	2003 Tax Ct. Memo LEXIS 279 *; T.C. Memo 2003-280; 86 T.C.M. (CCH) 412
30	3-Sep-2003	Lappo v. Comm'r	2	2003 Tax Ct. Memo LEXIS 257 *; T.C. Memo 2003-258; 86 T.C.M. (CCH) 333
31	20-Aug-2003	Hess v. Comm'r	1	2003 Tax Ct. Memo LEXIS 250 *; T.C. Memo 2003-251; 86 T.C.M. (CCH) 303
32	13-Jun-2003	Estate of Deputy v. Comm'r	1	2003 Tax Ct. Memo LEXIS 174 *; T.C. Memo 2003-176; 85 T.C.M. (CCH) 1497
33	14-May-2003	McCord v. Comm'r	1	120 T.C. 358 *; 2003 U.S. Tax Ct. LEXIS 16 **; 120 T.C. No. 13
34	23-Aug-2002	Okerlund v. United States	2	53 Fed. Cl. 341 *; 2002 U.S. Claims LEXIS 221 **; 2002-2 U.S. Tax Cas. (CCH) P60,447; 90 A.F.T.R.2d (RIA) 2002-6124
35	1-Aug-2002	Dunn v. Comm'r	1	301 F.3d 339 *; 2002 U.S. App. LEXIS 15453 **; 59 Fed. R. Serv. 3d (Callaghan) 529

#	Date	Case	Obs.	Reporter
36	17-Jun-2002	Estate of Bailey v. Comm'r	1	2002 Tax Ct. Memo LEXIS 159 *; T.C. Memo 2002-152; 83 T.C.M. (CCH) 1862; T.C.M. (RIA) 54788
37	9-Apr-2002	Estate of Mitchell v. Comm'r	1	2002 Tax Ct. Memo LEXIS 100 *; T.C. Memo 2002-98; 83 T.C.M. (CCH) 1524; T.C.M. (RIA) 54715
38	5-Feb-2002	Estate of Heck v. Comm'r	1	2002 Tax Ct. Memo LEXIS 38 *; T.C. Memo 2002-34; 83 T.C.M. (CCH) 1181; T.C.M. (RIA) 54639
39	3-Oct-2001	Estate of Elma Middleton Dailey v. Comm'r	2	2001 Tax Ct. Memo LEXIS 299 *; T.C. Memo 2001-263; 82 T.C.M. (CCH) 710
40	24-Aug-2001	Adams v. United States	1	2001 U.S. Dist. LEXIS 13092 *; 2001-2 U.S. Tax Cas. (CCH) P60,418; 88 A.F.T.R.2d (RIA) 2001-6057
41	6-Jul-2001	Estate of H.A. True v. Comm'r	9	2001 Tax Ct. Memo LEXIS 199 *; T.C. Memo 2001-167; 82 T.C.M. (CCH) 27
42	9-May-2001	Estate of Marcia P. Hoffman v. Comm'r	1	2001 Tax Ct. Memo LEXIS 136 *; T.C. Memo 2001-109; 81 T.C.M. (CCH) 1588
43	27-Mar-2001	Wall v. Comm'r	1	2001 Tax Ct. Memo LEXIS 97 *; T.C. Memo 2001-75; 81 T.C.M. (CCH) 1425
44	6-Mar-2001	Estate of Jones v. Comm'r	2	116 T.C. 121 *; 2001 U.S. Tax Ct. LEXIS 11 **; 116 T.C. No. 10; 116 T.C. No. 11
45	2-Feb-2001	Janda v. Comm'r	1	2001 Tax Ct. Memo LEXIS 34 *; T.C. Memo 2001-24; 81 T.C.M. (CCH) 1100; T.C.M. (RIA) 54231
46	30-Nov-2000	Knight v. Comm'r	1	115 T.C. 506 *; 2000 U.S. Tax Ct. LEXIS 88 **; 115 T.C. No. 36
47	18-Aug-2000	Estate of Borgatello v. Comm'r	1	2000 Tax Ct. Memo LEXIS 309 *; T.C. Memo 2000-264; 80 T.C.M. (CCH) 260; T.C.M. (RIA) 54013
48	4-Aug-2000	Godley v. Comm'r	4	2000 Tax Ct. Memo LEXIS 284 *; T.C. Memo 2000-242; 80 T.C.M. (CCH) 158; T.C.M. (RIA) 53984
49	27-Jun-2000	Estate of Klauss v. Comm'r	1	2000 Tax Ct. Memo LEXIS 228 *; T.C. Memo 2000-191; 79 T.C.M. (CCH) 2177; T.C.M. (RIA) 53923
50	11-Apr-2000	Estate of Maggos v. Comm'r	1	2000 Tax Ct. Memo LEXIS 154 *; T.C. Memo 2000-129; 79 T.C.M. (CCH) 1861
51	20-Mar-2000	Gow v. Comm'r	4	2000 Tax Ct. Memo LEXIS 108 *; T.C. Memo 2000-93; 79 T.C.M. (CCH) 1680
52	15-Feb-2000	Estate of Weinberg v. Comm'r	1	2000 Tax Ct. Memo LEXIS 58 *; T.C. Memo 2000-51; 79 T.C.M. (CCH) 1507
53	5-Nov-1999	Estate of Smith v. Comm'r	2	1999 Tax Ct. Memo LEXIS 425 *; T.C. Memo 1999-368; 78 T.C.M. (CCH) 745
54	14-Oct-1999	Estate of Marmaduke v. Comm'r	2	1999 Tax Ct. Memo LEXIS 397 *; T.C. Memo 1999-342; 78 T.C.M. (CCH) 590
55	23-Aug-1999	Estate of Hendrickson v. Comm'r	1	1999 Tax Ct. Memo LEXIS 318 *; T.C. Memo 1999-278; 78 T.C.M. (CCH) 322; T.C.M. (RIA) 99278
56	29-Jul-1999	Gross v. Comm'r	1	1999 Tax Ct. Memo LEXIS 290 *; T.C. Memo 1999-254; 78 T.C.M. (CCH) 201; T.C.M. (RIA) 99254
57	10-Mar-1999	Desmond v. Comm'r	1	1999 Tax Ct. Memo LEXIS 84 *; T.C. Memo 1999-76; 77 T.C.M. (CCH) 1529; T.C.M. (RIA) 99076
58	17-Nov-1998	Barnes v. Comm'r	2	1998 Tax Ct. Memo LEXIS 410 *; T.C. Memo 1998-413; 76 T.C.M. (CCH) 881; T.C.M. (RIA) 98413
59	8-Aug-1998	King v. Comm'r (Estate of Brookshire)	1	1998 Tax Ct. Memo LEXIS 370 *; T.C. Memo 1998-365; 76 T.C.M. (CCH) 659
60	30-Jun-1998	Estate of Davis v. Comm'r	1	110 T.C. 530 *; 1998 U.S. Tax Ct. LEXIS 35 **; 110 T.C. No. 35
61	30-Apr-1998	Furman v. Comm'r	2	1998 Tax Ct. Memo LEXIS 158 *; T.C. Memo 1998-157; 75 T.C.M. (CCH) 2206
62	19-Mar-1998	Dockery v. Comm'r	2	1998 Tax Ct. Memo LEXIS 114 *; T.C. Memo 1998-114; 75 T.C.M. (CCH) 2032
63	27-Oct-1997	Estate of Fleming v. Comm'r	1	1997 Tax Ct. Memo LEXIS 566 *; T.C. Memo 1997-484; 74 T.C.M. (CCH) 1049; 3 U.S. Tax Cas. (CCH) P45,035
64	5-Feb-1997	Gray v. Comm'r	1	1997 Tax Ct. Memo LEXIS 66 *; T.C. Memo 1997-67; 73 T.C.M. (CCH) 1940
65	26-Aug-1996	Estate of Barudin v. Comm'r	1	1996 Tax Ct. Memo LEXIS 403 *; T.C. Memo 1996-395; 72 T.C.M. (CCH) 488
66	11-Mar-1996	Kosman v. Comm'r	3	1996 Tax Ct. Memo LEXIS 107 *; T.C. Memo 1996-112; 71 T.C.M. (CCH) 2356
67	4-Dec-1995	Wheeler v. United States	1	1995 U.S. Dist. LEXIS 21432 *; 77 A.F.T.R.2d (RIA) 96-1405
68	7-Aug-1995	Estate of McCormick v. Comm'r	4	1995 Tax Ct. Memo LEXIS 367 *; T.C. Memo 1995-371; 70 T.C.M. (CCH) 318
69	12-Jun-1995	Mandelbaum v. Comm'r	1	1995 Tax Ct. Memo LEXIS 256 *; T.C. Memo 1995-255; 69 T.C.M. (CCH) 2852
70	28-Mar-1995	Estate of Frank v. Comm'r	1	1995 Tax Ct. Memo LEXIS 178 *; T.C. Memo 1995-132; 69 T.C.M. (CCH) 2255
71	27-Oct-1994	Estate of Luton v. Comm'r	2	1994 Tax Ct. Memo LEXIS 550 *; T.C. Memo 1994-539; 68 T.C.M. (CCH) 1044

#	Date	Case	Obs.	Reporter
72	19-Oct-1994	Estate of Lauder v. Comm'r	1	1994 Tax Ct. Memo LEXIS 535 *; T.C. Memo 1994-527; 68 T.C.M. (CCH) 985
73	11-May-1994	Estate of Simpson v. Comm'r	2	1994 Tax Ct. Memo LEXIS 217 *; T.C. Memo 1994-207; 67 T.C.M. (CCH) 2938
74	8-Dec-1993	Estate of Ford v. Comm'r	6	1993 Tax Ct. Memo LEXIS 595 *; T.C. Memo 1993-580; 66 T.C.M. (CCH) 1507
75	10-Nov-1993	Estate of Jung v. Comm'r	1	101 T.C. 412 *; 1993 U.S. Tax Ct. LEXIS 69 **; 101 T.C. No. 28
76	1-Feb-1993	Estate of Bennett v. Comm'r	1	1993 Tax Ct. Memo LEXIS 47 *; T.C. Memo 1993-34; 65 T.C.M. (CCH) 1816
77	30-Aug-1990	Estate of Murphy v. Comm'r	1	1990 Tax Ct. Memo LEXIS 520 *; T.C. Memo 1990-472; 60 T.C.M. (CCH) 645; T.C.M. (RIA) 90472
78	1-Aug-1990	Estate of Lenheim v. Comm'r	5	1990 Tax Ct. Memo LEXIS 420 *; T.C. Memo 1990-403; 60 T.C.M. (CCH) 356; T.C.M. (RIA) 90403
79	31-May-1990	Estate of Dougherty v. Comm'r	1	1990 Tax Ct. Memo LEXIS 292 *; T.C. Memo 1990-274; 59 T.C.M. (CCH) 772; T.C.M. (RIA) 90274
80	28-Feb-1990	Estate of Newhouse v. Comm'r	1	94 T.C. 193 *; 1990 U.S. Tax Ct. LEXIS 9 **; 94 T.C. No. 14
	TOTAL NUMBER OF OBSERVATIONS		**137**	

VII. EXHIBIT B: DESCRIPTIVE STATISTICS

A. Dependent Variable (DLOM)

N	Range	Minimum	Maximum	Mean	Std. Deviation
137	50.00	0.00	50.00	23.7461	8.86598

B. Independent Variables

Below we provide descriptive statistics for the independent variables. The cell labeled "missing" corresponds with the cases and observations in which information is unavailable or the independent reviewers provided conflicting conclusions.

Variable	N	%
TRA	62	45.26%
TRB	24	17.52%
TRC	41	29.93%
(missing)	10	7.30%
	137	100.00%

Variable	N	%
LIQA	109	79.569%
LIQB	19	13.87%
LIQC	8	5.84%
(missing)	1	0.73%
	137	100.00%

C. Control Variables

We report key statics for the continuous and categorical control variables below. The four missing values for the size of the interest correspond to the observations for which the independent reviewers could not determine the value in an unequivocal way.

The regression results have been checked on absence of multicollinearity. This situation occurs when the predictor variables are highly correlated with each other; in this case, the regression model would not be able to accurately associate variance in the outcome variable with the correct predictor variable, leading to muddled results and incorrect inferences. Multicollinearity has been checked using the variance inflation factor values (not reported).

Continuous variables	N	Minimum	Maximum	Mean	Std. Deviation
Company size ($M)	137	0.20	3141.98	120.2715	372.02089
Size of interest (%)	133	0.12	100.00	35.8352	28.62862
Spread (0–200)	137	3.28	200.00	64.2331	34.61747
DLOC (%)	137	.00	62.87	13.7915	9.27448

Categorical variables	1	0	Missing	Total (N)
Audit	22	115	0	137
Profitable	117	14	6	137

D. SIC Codes & Industry Classification

Division	Industry	SIC range	Number of observations
A	Agriculture, Forestry, & Fishing	01–09	7
B	Mining	10–14	2
C	Construction	15–17	4
D	Manufacturing	20–39	14
E	Transportation & Public Utilities	40–49	9
F	Wholesale Trade	50–51	5
G	Retail Trade	52–59	8
H	Finance, Insurance, & Real Estate	60–67	76
I	Services	70–89	12
J	Public Administration	91–98	0
K	Non classifiable Establishments	99	0
TOTAL			137